

Q10 Ampere's Law is the right one to use when there is a great deal of symmetry, making the circulation integral fairly easy and resulting in a simple expression in which B , the field at the desired location, appears. We can solve it for B without having to account for the contributions to the field from all the little current elements, which, though symmetric, might nonetheless result in a pretty difficult integral, for the current may be all over the place. It's of course nice to have a fall-back. If the symmetry isn't such that the circulation integral in Ampere's Law can be done, then we do have an alternative: the Biot-Savart Law.

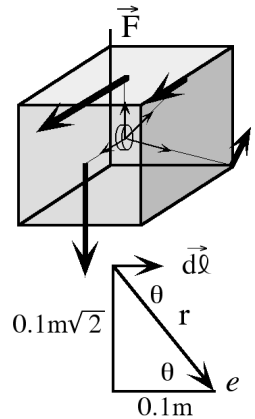
$$10 \quad \frac{\mu_0 I}{4\pi r^2} |\vec{d}\vec{\ell} \times \hat{r}| = \frac{(4\pi \times 10^{-7} \text{T}\cdot\text{m/A})(200 \text{A})}{4\pi r^2} (10^{-3} \text{m})(1) \sin \theta = \frac{2 \times 10^{-8} \text{T}\cdot\text{m}^2 (\sin \theta)}{r^2}$$

(a) $\frac{2 \times 10^{-8} \text{T}\cdot\text{m}^2 (\sin 90^\circ)}{(0.1 \text{m})^2} = 2 \times 10^{-6} \text{T}$. (b) $\frac{2 \times 10^{-8} \text{T}\cdot\text{m}^2 (\sin 45^\circ)}{(0.1 \text{m} \sqrt{2})^2} = 7.07 \times 10^{-7} \text{T}$

(c) $\frac{2 \times 10^{-8} \text{T}\cdot\text{m}^2 (\sin 90^\circ)}{(0.1 \text{m})^2} = 2 \times 10^{-6} \text{T}$. (d) $\frac{2 \times 10^{-8} \text{T}\cdot\text{m}^2 (\sin 0^\circ)}{(0.1 \text{m})^2} = 0$

(e) Here, $r^2 = (0.1 \text{m})^2 + (0.1 \text{m} \sqrt{2})^2 = 0.03 \text{m}^2$ and $\sin \theta = \frac{0.1 \text{m} \sqrt{2}}{r} = 0.816$

$$\frac{2 \times 10^{-8} \text{T}\cdot\text{m}^2 (0.816)}{0.03 \text{m}^2} = 5.44 \times 10^{-7} \text{T}$$



36 (a) Zero, for no current is enclosed. (b) $-I_1$ is enclosed—negative because it is opposite the direction of my right thumb (with my fingers, like the path, counterclockwise). Thus $\oint \vec{B} \cdot \vec{d}\vec{\ell} = \mu_0 (-4A)$
 (c) $+I_2 - I_1$ is enclosed. $\oint \vec{B} \cdot \vec{d}\vec{\ell} = \mu_0 (+2A)$. (d) $+I_3 + I_2 - I_1$ is enclosed. $\oint \vec{B} \cdot \vec{d}\vec{\ell} = \mu_0 (+4A)$

41 $B = \mu_0 n I \rightarrow 0.027 \text{T} = (4\pi \times 10^{-7} \text{T}\cdot\text{m/A}) n (12 \text{A}) \Rightarrow n = 1790 \text{m}^{-1}$. (b) $(1790 \text{m}^{-1})(0.4 \text{m}) = 716$ turns.
 $716 \text{turn} \cdot 2\pi(0.014 \text{m}) = 63 \text{m}$

44 $B = \frac{\mu_0 N I}{2\pi r}$ inside. (b) $B = \frac{(4\pi \times 10^{-7} \text{T}\cdot\text{m/A})(250)(8.5 \text{A})}{2\pi(0.16 \text{m})} = 2.66 \times 10^{-3} \text{T}$. (a) and (c) Both of these are outside the toroid. As noted in the text, the field outside a toroid is very small.

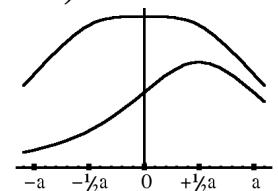
64 The field due to the wire is $\frac{\mu_0(14 \text{A})}{2\pi r}$ and exerts a force on a segment of $I \vec{\ell} \times \vec{B} = (5 \text{A}) \ell \frac{\mu_0(14 \text{A})}{2\pi r} \sin \theta$.
 Actually, the upward force on the top segment and downward at bottom cancel, leaving an inward attraction on the left segment and outward repulsion on the right. $F_{\text{attract}} = (5 \text{A})(0.2 \text{m}) \frac{\mu_0(14 \text{A})}{2\pi(0.026 \text{m})} = 1.08 \times 10^{-4} \text{N}$.
 $F_{\text{repel}} = (5 \text{A})(0.2 \text{m}) \frac{\mu_0(14 \text{A})}{2\pi(0.1 \text{m})} = 2.8 \times 10^{-5} \text{N}$. Net attraction is $8.0 \times 10^{-5} \text{N}$

67 Equation 28.15 is correct for the axis of a loop, but where x is measured from the center. We need to shift this over $+1/2a$ for one coil, and $-1/2a$ for the other. $B = \frac{\mu_0 N I a^2}{2((x-1/2a)^2 + a^2)^{3/2}} + \frac{\mu_0 N I a^2}{2((x+1/2a)^2 + a^2)^{3/2}}$

(b) The two-coil field is a lot flatter in the region between the coils.

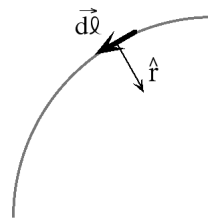
(c) $\frac{\mu_0(300)(6 \text{A})(0.08 \text{m})^2}{2((0-1/2(0.08 \text{m}))^2 + (0.08 \text{m})^2)^{3/2}} + \frac{\mu_0(300)(6 \text{A})(0.08 \text{m})^2}{2((0+1/2(0.08 \text{m}))^2 + (0.08 \text{m})^2)^{3/2}} = 0.020 \text{T}$

Note: The problem says “very uniform magnetic field in the region between”. A long solenoid produces a “very” uniform field. The field of Helmholtz coils is not quite as uniform, but still very good, and allows easy access inside (as in lab).



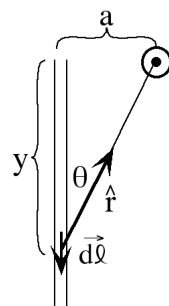
69] Ampere's Law won't do, for there is no useful path along which to calculate $\oint \vec{B} \cdot d\vec{\ell}$.

Thus, use $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$. The straight segments can be ignored, because the \hat{r} from these sources to where the field is being found give a zero value for $d\vec{\ell} \times \hat{r}$. For the curved section, all the $d\vec{\ell} \times \hat{r}$ are out of the page, so we can just add magnitudes. Also, $r = R$ the way, and $|d\vec{\ell} \times \hat{r}| = d\ell \sin 90^\circ = d\ell$. Thus $dB = \frac{\mu_0 I d\ell}{4\pi R^2}$. $B = \int dB = \frac{\mu_0 I}{4\pi R^2} \int d\ell = \frac{\mu_0 I}{4\pi R^2} \frac{\pi R}{2} = \frac{\mu_0 I}{8R}$



70] Here too, Ampere's Law won't do. The horizontal segment gives no field at P, for the same reason as straight segments in prob. 69. All the $d\vec{\ell} \times \hat{r}$ for the vertical segment are out of the page, so just add magnitudes. $B = \int dB$. Choosing y for my integration variable, $r = \sqrt{a^2 + y^2}$, and $|d\vec{\ell} \times \hat{r}| = dy \sin \theta$ (actually, it's $180^\circ - \theta$, but the sine is the same) and

this becomes $dy \frac{a}{\sqrt{a^2 + y^2}}$. Thus $\int dB = \int_0^\infty \frac{\mu_0 I}{4\pi} \frac{dy}{a^2 + y^2} \frac{a}{\sqrt{a^2 + y^2}} = \frac{\mu_0 I}{4\pi a}$



72] In other words, find B everywhere. This is a problem where adding the contribution to the field from all the current elements everywhere would be a terrific headache. But there is sufficient symmetry to apply Ampere's Law. I'm gonna save time. I assume \vec{B} is counterclockwise everywhere, and I'm going to integrate counterclockwise around a circular amperian loop of radius r . Just like in class, $\oint \vec{B} \cdot d\vec{\ell} = \oint B d\ell \cos 0^\circ = \oint B d\ell = B \oint d\ell = B 2\pi r$. Great! This is done---*finished*---for all four "regions". Now, the other "half":

For $r < a$, $I_{\text{encl}} = +I \frac{\pi r^2}{\pi a^2}$ because my amperian loop, when in this region, encloses (1) current in the direction of my right thumb with my fingers in the counterclockwise direction of my path, and thus positive, and (b) a fraction of the total current in this inner conductor equal to the fraction of the total *area* enclosed.

For $a < r < b$, $I_{\text{encl}} = +I$ for all this positive current is enclosed.

For $b < r < c$, $I_{\text{encl}} = +I - I \frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2}$ because my loop encloses all the inner positive current, plus a fraction of the negative (opposite direction) current in the outer conductor equal to the fraction of its total cross-sectional area enclosed. This

simplifies to $I_{\text{encl}} = I \frac{c^2 - r^2}{c^2 - b^2}$

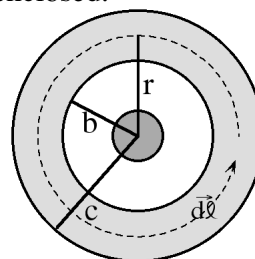
For $r > c$, $I_{\text{encl}} = +I - I = 0$.

Now, putting them together: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}}$

$r < a$: $B 2\pi r = \mu_0 I \frac{r^2}{a^2} \Rightarrow B = \frac{\mu_0 I r}{2\pi a^2}$

$a < r < b$: $B 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$ (agrees at $r = a$)

$b < r < c$: $B 2\pi r = \mu_0 I \frac{c^2 - r^2}{c^2 - b^2} \Rightarrow B = \frac{\mu_0 I}{2\pi r} \frac{c^2 - r^2}{c^2 - b^2}$ (agrees at $r = b$) $r > c$: $B 2\pi r = 0 \Rightarrow B = 0$ (agrees at $r = c$)



73] If it did, there would be a net outward flux, but $\oint \vec{B} \cdot d\vec{A}$ must be zero, for there are no monopoles.

80] $\oint \vec{B} \cdot d\vec{\ell} = \int_{\text{bottom}} \vec{B} \cdot d\vec{\ell} + \int_{\text{top}} \vec{B} \cdot d\vec{\ell} + \int_{\text{sides}} \vec{B} \cdot d\vec{\ell}$. The first integral is just BL.

The second, along the top, is zero, because, by assumption, the field is zero there.

The third is zero because the path is \perp to \vec{B} . Thus, $\oint \vec{B} \cdot d\vec{\ell} = BL$. *But this path encloses no current*, so the circulation integral has to be zero. The contradiction.

