$\boxed{15}$ C_1 and C_2 are in series, giving an equivalent capacitance of ½ 4μF = 2μF. This is in parallel with another 4μF, giving 6μF, which in turn is in series with another 4μF. $C_{eq} = \left(\frac{1}{4\mu F} + \frac{1}{6\mu F}\right)^{-1}$ = 2.4μF. Therefore the charge is $Q = CV = (2.4\times10^{-6}F)(28V) = 6.72\times10^{-5}C$. This must be the charge on both of the capacitors, C_4 and the combination of the others, in series across the source. If C_3 and the C_1–C_2 combination are in parallel, they have the same potential difference, $\Delta V = Q/C$. So C_3, with twice the capacitance, will have twice the charge as on the 2μF C_1–C_2 combination. $\frac{2}{3} \times 6.72\times10^{-5}C = 4.48\times10^{-5}C$ is on C_3 while $\frac{1}{3} \times 6.72\times10^{-5}C = 2.24\times10^{-5}C$ is left for the combination. Since they're in series, this must be the charge on each.
(b) $\Delta V_1 = Q_1/C_1 = (2.24\times10^{-5}C)/(4\mu F) = 5.6V$. $\Delta V_2 = Q_2/C_2$ = same. $\Delta V_3 = Q_3/C_3$, or it must simply be the sum of ΔV_1 and ΔV_2, 11.2V. $\Delta V_4 = Q_4/C_4 = (6.72\times10^{-5}C)/(4\mu F) = 16.8V$. (c) The potential difference between *a* and *d* is just that across C_3, 11.2V

$\boxed{26}$ $C = \frac{Q}{\Delta V} = \frac{1.8\times10^{-8}C}{200} = 9\times10^{-11}F$. (b) $C = \epsilon_o \frac{A}{d} \rightarrow 9\times10^{-11}F = 8.85\times10^{-12}C^2/N\cdot m^2 \frac{A}{1.5\times10^{-3}m}$
$\Rightarrow A = 0.0153m^2$. (b) $\Delta V = \frac{\text{volts}}{\text{meter}}$ meters $= (3\times10^6V/m)(0.0015m) = 4500V$.
(d) $\frac{1}{2}Q^2/C = \frac{1}{2}(1.8\times10^{-8}C)^2/(9\times10^{-11}F) = 1.8\times10^{-6}J$

$\boxed{50}$ $C = \epsilon_o \frac{A}{d} = 8.85\times10^{-12}C^2/N\cdot m^2 \frac{(0.16m)^2}{4.7\times10^{-3}m} = 4.82\times10^{-11}F$. (b) $Q = C\Delta V = (4.82\times10^{-11}F)(12V) = 5.78\times10^{-10}C$ (c) $\frac{\text{volts}}{\text{meter}} = \frac{12V}{0.0047m} = 2.55\times10^3 N/C$. (d) $\frac{1}{2}C\Delta V^2 = \frac{1}{2}(4.82\times10^{-11}F)(12V)^2 = 3.47\times10^{-9}J$
(a) The capacitance will be half, $2.41\times10^{-11}F$. Since the charge has nowhere to go, it remains constant, $5.78\times10^{-10}C$. The field due to a large plate is uniform, so as long as the charge doesn't change, the field is the same. (Note: A constant Q while C is only half as large $\Rightarrow \Delta V = Q/C$ must double. This fits: Twice the ΔV over twice the distance $\Rightarrow$ equal field.) $U = \frac{1}{2}Q^2/C$, with Q constant and C being half, must go up by a factor of 2, to $6.94\times10^{-9}J$. Of course this energy must come from whoever pulls the plates apart.

$\boxed{51}$ The capacitance will again be half, $2.41\times10^{-11}F$. Since ΔV is the same, Q must be half (some goes back to the battery), $2.89\times10^{-10}C$. Same volts, but per twice the distance, so field is half (which makes sense if there is half the charge), $1.28\times10^3N/C$. Same ΔV but half the C $\Rightarrow$ half the energy, $1.74\times10^{-9}J$. Now it's a very good question how this happens. You'd have to pull the plates apart, adding energy, but the energy stored in the capacitor *goes down*. The battery is being "charged", storing quite a bit of energy.

$\boxed{55}$ $C = \frac{2\pi\epsilon_o L}{\ln[(r_a+d)/r_a]} = \frac{2\pi\epsilon_o L}{\ln[1+d/r_a]} \cong \frac{2\pi\epsilon_o L}{d/r_a} = \epsilon_o \frac{2\pi r_a L}{d} = \epsilon_o \frac{A}{d}$. (b) Just like in calculus, the thickness is so small that it's essentially that same area A at the outside and inside surfaces of the cylindrical shell.

$\boxed{56}$ $Q_1 = C_1\Delta V = (9\times10^{-6}\text{F})(28\text{V}) = 2.52\times10^{-4}\text{C}$. $Q_2 = (4\times10^{-6}\text{F})(28\text{V}) = 1.12\times10^{-4}\text{C}$.
After the swap there will be one pair of connected plates, the new positive plates, sharing a charge of $(+2.52\times10^{-4}\text{C}) + (-1.12\times10^{-4}\text{C}) = +1.4\times10^{-4}\text{C}$ and the other pair of connected plates sharing $-1.4\times10^{-4}\text{C}$. The positive will distribute itself proportionately between the capacitors' new positive plates, and the negative between their new negative plates, because the capacitors eventually must end up with the same (new) potential difference. Calling the charge that ends up on the positive plate of the 9μF Q´, the charge on the connected plate of the 4μF would be $1.4\times10^{-4}\text{C} - Q'$. Setting their potential difference, Q/C, equal, $\frac{Q'}{9\mu\text{F}} = \frac{1.4\times10^{-4}\text{C} - Q'}{4\mu\text{F}}$. Solving, $Q' = 9.69\times10^{-5}\text{C}$, so that the 4μF has $4.31\times10^{-5}\text{C}$. Using $U = \frac{1}{2}Q^2/C$, the energy stored is $U_{1\text{new}} + U_{2\text{new}} = \frac{1}{2}(9.69\times10^{-5}\text{C})^2/(9\times10^{-6}\text{F}) + \frac{1}{2}(4.31\times10^{-5}\text{C})^2/(4\times10^{-6}\text{F}) = 7.5\times10^{-4}\text{J}$. Originally, $U_{1\text{old}} + U_{2\text{old}} = \frac{1}{2}(2.52\times10^{-4}\text{C})^2/(9\times10^{-6}\text{F}) + \frac{1}{2}(1.12\times10^{-4}\text{C})^2/(4\times10^{-6}\text{F}) = 5.1\times10^{-3}\text{J}$. An energy of 4.34×10^{-4}J is lost. Where'd it go?! The first law of thermodynamics says that energy is conserved. But this is an irreversible process. If an *incremental* change cannot effect a reversal of a process (and you can't incrementally connect a wire), that is, if it is spontaneous, then the process is irreversible. It will necessarily be accompanied by an increase in disorder (Entropy) and the conversion of mechanical energy (kinetic plus potential) to thermal energy. The moving charges *will* lose energy to heat. Exactly how is a different matter. If there is resistance (a good bet), it will become heat there. Were resistance negligible, once the connection is made, there would be charges oscillating back and forth between the two capacitors. Oscillating charges generate electromagnetic waves, thus shedding the energy.

$\boxed{67}$ Well, a charge residing on a conductor will give it a certain potential relative to infinity, and whether or not there is another plate with opposite charge at infinity wouldn't really change things. (b) Assume a positive charge Q on the sphere. The field, by Gauss' law, is $\frac{kQ}{r^2}$.

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{\ell} \;\rightarrow\; V_\infty - V_R = -\int_R^\infty \frac{kQ}{r^2}\,dr = -\frac{kQ}{R}. \quad C = \frac{Q}{\Delta V} = \frac{Q}{kQ/R} = \frac{R}{k} = 4\pi\epsilon_o R$$

(c) $\frac{R}{k} = \frac{6.37\times10^6\text{m}}{9\times10^9\text{N}\cdot\text{m}^2/\text{C}^2} = 7.1\times10^{-4}\text{F}$. Pretty big!

$\boxed{70}$ $u = \frac{1}{2}\epsilon_o E^2$. By now we know that the field will be just like that of a wire on the axis: $E = \frac{\lambda}{2\pi\epsilon_o r}$

Thus, $u = \frac{1}{2}\epsilon_o \frac{\lambda^2}{4\pi^2\epsilon_o{}^2 r^2} = \frac{\lambda^2}{8\pi^2\epsilon_o r^2}$. We must multiply the energy density u by a volume where it's constant, a cylindrical shell, $dV = 2\pi rL\,dr$, then add them up. $U = \int u\,dV = \int_{r_a}^{r_b} \frac{\lambda^2}{8\pi^2\epsilon_o r^2}(2\pi rL\,dr) = \frac{\lambda^2 L}{4\pi\epsilon_o}\ln(r_b/r_a)$

Thus, $U/L = \frac{\lambda^2}{4\pi\epsilon_o}\ln(r_b/r_a)$ (c) $U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(\lambda L)^2}{2\pi\epsilon_o L/\ln(r_b/r_a)} = \frac{\lambda^2 L}{4\pi\epsilon_o}\ln(r_b/r_a)$, giving the same U/L. These have to agree, for we interpret the energy as really being in the field.