Q6 $-\int \vec{E}\cdot d\vec{\ell} = \Delta V$. If $\vec{E}$ is zero, it means that ΔV is zero, meaning V doesn't *change*––that's all we can say. We *choose* the place where V is zero, and the region in question may or may not be that place.

Q10 $\vec{E} = -\vec{\nabla}V$. A gradient is a derivative. Unless we know how it *changes* in the region, we can't find $\vec{E}$.

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27 The difference in potential energy per unit charge is given. Therefore, to find the difference in the potential energy, multiply by the charge. $\Delta U = q_o\,\Delta V = (-1.6\times10^{-19}C)(295V-0V) = -4.72\times10^{-17}J$. The final kinetic energy is this magnitude. $4.72\times10^{-17}J = \frac{1}{2}(9.11\times10^{-31}kg)v^2 \Rightarrow v = 1.0\times10^7 m/s$

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29 $\vec{E} = -\vec{\nabla}V \Rightarrow$ V increases opposite the field. $\vec{E}$ is in the negative direction, so V_b must be higher.

(b) $|E| = |\frac{d}{dx}V| = \frac{240V}{0.3m} = 800N/C$. (c) $\Delta U = q_o\,\Delta V = (-0.2\times10^{-6}C)(-240V) = 4.8\times10^{-5}J$. The force on the negative is opposite the field, so it is in the positive direction. Thus it does negative work and the potential energy increases. Work $= -4.8\times10^{-5}J$. Or $\vec{F}\cdot d\vec{\ell} = |F|\,|d\ell| \cos\theta = [(0.2\times10^{-6}C)(800N/C)](0.3m)(-1)$

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48 $\vec{E} = -\vec{\nabla}V \Rightarrow E_x = -\frac{\partial}{\partial x}V = \frac{xQ}{4\pi\epsilon_o(x^2+y^2+z^2)^{3/2}}$. Similarly, $E_y = \frac{yQ}{4\pi\epsilon_o(x^2+y^2+z^2)^{3/2}}$

$E_z = \frac{zQ}{4\pi\epsilon_o(x^2+y^2+z^2)^{3/2}}$. Thus, $\vec{E} = E_x\,\hat{i} + E_y\,\hat{j} + E_z\,\hat{k} = \frac{Q}{4\pi\epsilon_o(x^2+y^2+z^2)^{3/2}}(x\,\hat{i} + y\,\hat{j} + z\,\hat{k})$. But

$(x\,\hat{i} + y\,\hat{j} + z\,\hat{k})$ is just $\vec{r}$, or $r\,\hat{r}$. Thus, $\vec{E} = \frac{Q}{4\pi\epsilon_o(x^2+y^2+z^2)^{3/2}}\,r\,\hat{r}$. But $\sqrt{x^2+y^2+z^2} = r$, so $\vec{E} = \frac{Q}{4\pi\epsilon_o r^2}\hat{r}$

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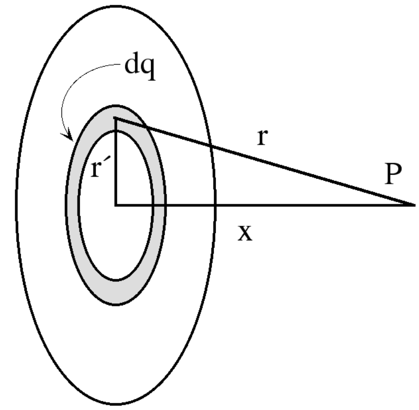
57 There are six pairs of negative–negative potential energy at a separation of $\sqrt{2}$ d, and six pairs of positive–positive potential energy at a separation of $\sqrt{2}$ d. Because U is positive, corresponding to a repulsive force, if charges are of the same sign, these all have the same value. $U_{repel} = 12\times\frac{k\,qq}{r} = 12\times\frac{k\,q^2}{\sqrt{2}d} = 8.49\,\frac{k\,q^2}{d}$

Each positive attracts each negative. There are 12 attractions at a separation of d and 4 at $\sqrt{3}$ d.

$U_{attract} = 12\times\frac{k\,(+q)(-q)}{d} + 4\times\frac{k\,(+q)(-q)}{\sqrt{3}d} = -14.31\,\frac{k\,q^2}{d}$. $U_{total} = (-14.31+8.49)\,\frac{k\,q^2}{d} = -5.82\,\frac{k\,q^2}{d}$

(b) Well, the overall potential energy is negative, which means that to pull everything apart, you'd have to put in positive energy. Conversely, it is a lower energy state for these atoms to be in a crystal, rather than separate.

66 All the charge in the annular region shown is the same distance from the point P, so it all would produce the same potential there. Thus, this region is good for dq. I choose r´ for my variable of integration. *As is always the case*, the d(whatever) is *never just "thrown in"* to the integral; *it comes from the dq*, through a relationship between charge and that variable, be it a distance, or an angle, or, as here, an area.



$$dq = \frac{\text{charge}}{\text{area}}\, d(\text{area}) = \sigma\,(2\pi r'\,dr') \qquad r = \sqrt{r'^2+x^2}$$

(You won't see sine or cosine below. *Potential is a scalar!*)

$$V = \int dV = \int \frac{k\,dq}{r} = \int_0^R \frac{k\,\sigma(2\pi r'\,dr')}{\sqrt{r'^2+x^2}} = 2\pi\, k\, \sigma \sqrt{r'^2+x^2}\,\Big|_0^R$$

$$= 2\pi\, k\, \sigma\,(\sqrt{R^2+x^2} - x). \text{ (b) } -\frac{\partial}{\partial x} 2\pi\, k\, \sigma\,(\sqrt{R^2+x^2} - x) = -2\pi\, k\, \sigma\left(\frac{x}{\sqrt{R^2+x^2}} - 1\right) = \frac{1}{2\epsilon_o}\,\sigma\left(1 - \frac{x}{\sqrt{R^2+x^2}}\right).$$

This shows obtaining E_x from a derivative of V. The reverse is obtaining ΔV by integrating $\vec{E}$.

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69 I choose to integrate in the positive direction along the x axis (the only place where I know E), using an x´ for my integration variable, from the point x´ = x to the point x´ = ∞, where I know the potential to be zero.

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{\ell} \;\rightarrow\; V_\infty - V_x = -\int_x^\infty E(x')\,dx'\cos 0^\circ = -\int_x^\infty k\,\frac{Q\,x}{(x'^2 + a^2)^{3/2}}\,dx' = \frac{kQ}{\sqrt{x'^2+a^2}}\,\Big|_x^\infty = -\frac{kQ}{\sqrt{x^2+a^2}}$$

Setting $V_\infty = 0$, we have $0 - V_x = -\dfrac{kQ}{\sqrt{x^2+a^2}}$ or $V_x = \dfrac{kQ}{\sqrt{x^2+a^2}}$ (agrees)

You might think it more natural to integrate from ∞ to x (rather than from x to ∞), from where we know the potential to where we hope to find it, and you may thus find it puzzling that in all examples in the chapter the actual integration is always in the positive direction. It is wise to do this, for it is very easy to make a sign error when doing a line integral in the negative direction.

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85 KE → PE, or $2 \times \frac{1}{2}mv^2 \rightarrow \dfrac{kqq}{\text{diameter}}$ or $2 \times \frac{1}{2}(1.67\times10^{-27}\text{kg})\, v^2 = \dfrac{(9\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\text{C})^2}{2(1.2\times10^{-15}\text{m})}$

$\Rightarrow v = 7.6\times10^6$m/s. (b) $2 \times \frac{1}{2}(2.99\times1.67\times10^{-27}\text{kg})\, v^2 = \dfrac{(9\times10^9\text{N}\cdot\text{m}^2/\text{C}^2)(2\times1.6\times10^{-19}\text{C})^2}{3.5\times10^{-15}\text{m}}$

$\Rightarrow v = 7.3\times10^6$m/s. (c) $\frac{3}{2}(1.38\times10^{-23}\text{J/K})\, T = \frac{1}{2}(1.67\times10^{-27}\text{kg})\,(7.6\times10^6\text{m/s})^2 \Rightarrow T = 2.3\times10^9\text{K}$

$\frac{3}{2}(1.38\times10^{-23}\text{J/K})\, T = \frac{1}{2}(2.99\times1.67\times10^{-27}\text{kg})\,(7.3\times10^6\text{m/s})^2 \Rightarrow T = 6.4\times10^9\text{K}$. (d) The sun's temperature would seem too low, but there will always be particles moving faster than the rms speed––and some don't have to actually surmount the electrostatic barrier, but can "tunnel" through quantum–mechanically.