Homework Three, Physics 242, Spring 2009
Due Wednesday, April 29

[**1.**] Consider

$$H = -t \sum_l (c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l) + G \sum_l (-1)^l c_l^\dagger c_l \ .$$

This corresponds to a model of a one dimensional solid where the nucleii (lattice sites) alternate between two types "A" and "B", with energies $\pm G$.

Show, as in the case of the BCS Hamiltonian, that going to momentum space does not fully diagonalize H. What states remain mixed? Define an appropriate final rotation (linear combination of the mixed states) to complete the diagonalization. What is the dispersion relation? Sketch it. What are the allowed k values after the final rotation?

[**2.**] In class we went through the details of the Bogliubov-Vatutin transformation of the $\sum_k \epsilon_k c_{k\sigma}^\dagger c_{k\sigma}$ piece of the BCS Hamiltonian, but merely wrote down the result for the pairing term. Work through the algebra of the pairing term and derive the answer asserted in class.

[**3.**] In class, we similarly skipped the algebra which showed the coefficient of the "good" terms obeys,

$$\epsilon_k (u_k^2 - v_k^2) + 2\Delta_k u_k v_k = E_k \ .$$

Derive this result.