

What Does Tossing a Coin Have to do with the Binomial Theorem?

[1] If you toss a coin once, the possible outcomes are H and T where $H =$ heads and $T =$ tails. If you toss the coin twice, four things can occur: HH, HT, TH, TT . List all the possible choices for tossing a coin 4 times.

[2] In your algebra course you learned $(a + b)^2 = a^2 + 2ab + b^2$. Let's write this in a bit more complicated way as $(a + b)^2 = a \cdot a + a \cdot b + b \cdot a + b \cdot b$ where we have avoided using the power notation for $a^2 = a \cdot a$ and have also refused to combine $a \cdot b$ with $b \cdot a$. Write down $(a + b)^3$ and $(a + b)^4$ in this more complex way. It is best if you not only refuse to combine terms but also refrain from re-ordering the products using the commutative law. That is, if you have a term $a \cdot b \cdot a$, leave it in that order.

[3] Compare your answers to [1] and [2]. The comparison illustrates that there is a connection between coin-tossing and the binomial theorem.